The Universal Invariant of Physical Reality

Recovery and normalization uniquely determine σ, with exact scale invariance, dimensional self-closure, and spherical closure.

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ABSTRACT

Analytic structure, recovery, and closure

The recovery identities and integral normalization determine one σ uniquely on $I = (-1/\gamma, \infty)$, where $\mu > 0$ and $\gamma > \mu$. Its explicit differential hierarchy, exact scale invariance, transform recovery, dimensional self-closure, and spherical closure follow simultaneously. The Laplace transform returns μ and γ globally, and exact Lambert inversion has two real branches that meet at the unique closure.

Dimensional self-closure is the absence of a residual inverse-square term. It selects $D = 3$ uniquely among integer spatial dimensions $D \geq 2$. In three dimensions, the sphere $x_1^2 + x_2^2 + x_3^2 = 1/\mu - 1/\gamma$ is the complete closure set. No amplitude, finite differential data, scale prescription, spatial dimension under dimensional self-closure, or closure geometry remains independently specifiable.

CITATION

A. Albert (2026)

Albert, A. (2026). The Universal Invariant of Physical Reality. Mathematical Research Institute of Physical Reality. https://www.mripr.org/en/research/the-universal-invariant-of-physical-reality/

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@misc{albert2026universalinvariant,
  author      = {Albert, Alex},
  title       = {The Universal Invariant of Physical Reality},
  year        = {2026},
  publisher   = {Mathematical Research Institute of Physical Reality},
  address     = {Pécs, Hungary},
  url         = {https://www.mripr.org/en/research/the-universal-invariant-of-physical-
reality/},
  note        = {Published 25 July 2026}
}
```

MATHEMATICAL CORE

One invariant, in logical order

Every statement and proof below concerns the same σ. The numbered separation records proof dependencies and introduces no independent objects. The final theorem collects the normalization, recovery, rigidity, scale, dimensional, and spatial results proved by the sequence.

1 Definition and domain

Let

$$\mu > 0, \qquad \gamma > \mu,$$

and define

$$I = \left(-\frac{1}{\gamma}, \infty\right).$$

For $r \in I$, let

$$\sigma(r) = \ln\left(\frac{\mu^2}{\mu+\gamma}\right) + \ln(1+\gamma r) - \mu r. \tag{1}$$

Its exponential representation is

$$e^{\sigma(r)} = \frac{\mu^2}{\mu+\gamma}(1+\gamma r)e^{-\mu r}. \tag{2}$$

Equation (2) follows directly from (1), and (1) is recovered from (2) by the real logarithm.

$$\mu > 0$$

gives convergence of the normalization integral.

$$\gamma > 0$$

places $[0, \infty)$ inside the logarithmic domain, and

$$\gamma > \mu \quad\Longleftrightarrow\quad r_* = \frac{1}{\mu} - \frac{1}{\gamma} > 0.$$

The condition $\gamma > \mu$ places closure at a positive spatial radius.

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2 Normalization

PROPOSITION 1

The exponential representation is normalized on $[0, \infty)$:

$$\int_0^\infty e^{\sigma(r)}\,dr = 1. \tag{3}$$

The prefactor $\mu^2/(\mu + \gamma)$ is the unique normalizing factor. No independent amplitude remains.

PROOF.

Since $\mu > 0$,

$$\int_0^\infty e^{-\mu r}\,dr = \frac{1}{\mu}$$

and

$$\int_0^\infty r e^{-\mu r}\,dr = \frac{1}{\mu^2}.$$

Combining these integrals gives

$$\begin{aligned}\int_0^\infty e^{\sigma(r)}\,dr &= \frac{\mu^2}{\mu+\gamma}\int_0^\infty (1+\gamma r)e^{-\mu r}\,dr \\ &= \frac{\mu^2}{\mu+\gamma}\left(\frac{1}{\mu}+\frac{\gamma}{\mu^2}\right) \\ &= 1.\end{aligned}$$

Also,

$$\int_0^\infty (1+\gamma r)e^{-\mu r}\,dr = \frac{\mu+\gamma}{\mu^2}.$$

Its reciprocal is $\mu^2/(\mu + \gamma)$, so no other constant normalizes the expression.

Q.E.D.

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3 Differential hierarchy

PROPOSITION 2

The logarithmic representation σ is real analytic on I. Its first two derivatives with respect to r are

$$\sigma'(r) = \frac{\gamma}{1+\gamma r} - \mu \tag{4}$$

and

$$\sigma''(r) = -\frac{\gamma^2}{(1+\gamma r)^2}. \tag{5}$$

For every integer $n \geq 2$ and every $r \in I$,

$$\sigma^{(n)}(r) = (-1)^{n-1}(n-1)!\frac{\gamma^n}{(1+\gamma r)^n}. \tag{6}$$

PROOF.

Equation (4) follows by differentiating (1). Differentiating (4) gives (5). Repeated differentiation of $\ln(1 + \gamma r)$ gives (6). The constant and linear terms vanish after the first derivative.

Q.E.D.

COROLLARY 1

The logarithmic representation is strictly concave with respect to r on I. The exponential representation is strictly log-concave and need not be concave.

PROOF.

Equation (5) gives

$$\sigma''(r) < 0$$

for every $r \in I$. Since $\ln(e^{\sigma(r)}) = \sigma(r)$, the exponential representation is strictly log-concave. Its second derivative is

$$\frac{d^2}{dr^2} e^{\sigma(r)} = e^{\sigma(r)} \left[(\sigma'(r))^2 + \sigma''(r) \right] . \tag{7}$$

The bracket equals $\mu^2 - 2\mu\gamma/(1 + \gamma r)$. It is negative at r_* and tends to $\mu^2 > 0$ as r tends to infinity, so it has no fixed sign on I.

Q.E.D.

PROPOSITION 3

For every integer $n \geq 3$ and every $r \in I$,

$$(1+\gamma r)\sigma^{(n)}(r) + (n-1)\gamma\sigma^{(n-1)}(r) = 0. \tag{8}$$

For every $r \in I$, the second-order relation is

$$(1+\gamma r)\sigma''(r) + \gamma(\sigma'(r) + \mu) = 0. \tag{9}$$

PROOF.

Substitute (6) into the left-hand side of (8). The two terms cancel. Equation (9) follows from

$$\sigma'(r) + \mu = \frac{\gamma}{1+\gamma r}$$

and (5).

Q.E.D.

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4 Closure

THEOREM 1

The logarithmic representation has one stationary point,

$$r_* = \frac{1}{\mu} - \frac{1}{\gamma} = \frac{\gamma - \mu}{\mu\gamma}. \tag{10}$$

It is the unique global maximum on I.

PROOF.

By (4), the stationary equation is

$$\frac{\gamma}{1+\gamma r} = \mu.$$

Solving gives (10). Since $\gamma > \mu > 0$,

$$r_* > 0.$$

Strict concavity implies that there is at most one stationary point and that any stationary point is a strict maximum. At the two ends of I,

$$\lim_{r \to -1/\gamma^+} \sigma(r) = -\infty, \qquad \lim_{r \to \infty} \sigma(r) = -\infty.$$

The maximum is global.

Q.E.D.

At closure,

$$1 + \gamma r_* = \frac{\gamma}{\mu}, \tag{11}$$

so

$$\sigma''(r_*) = -\mu^2. \tag{12}$$

For $n \geq 2$, equation (6) gives

$$\sigma^{(n)}(r_*) = (-1)^{n-1}(n-1)!\mu^n. \tag{13}$$

The value at closure is

$$\sigma(r_*) = \ln\left(\frac{\mu\gamma}{\mu+\gamma}\right) - 1 + \frac{\mu}{\gamma}, \tag{14}$$

and therefore

$$e^{\sigma(r_*)} = \frac{\mu\gamma}{\mu+\gamma}e^{-1+\mu/\gamma}. \tag{15}$$

COROLLARY 2

The exponential representation has the same unique maximizing value r_*.

PROOF.

Since $e^{\sigma(r)} > 0$,

$$\frac{d}{dr}e^{\sigma(r)} = e^{\sigma(r)}\sigma'(r).$$

Its stationary points are the stationary points of σ. The exponential function preserves the ordering of real numbers.

Q.E.D.

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5 Recovery

THEOREM 2

For every $r \in I$,

$$\gamma = \frac{\sqrt{-\sigma''(r)}}{1 - r\sqrt{-\sigma''(r)}} \tag{16}$$

and

$$\mu = -\sigma'(r) + \sqrt{-\sigma''(r)}. \tag{17}$$

The identities recover μ and γ from σ itself at every $r \in I$.

PROOF.

Equation (5), together with $\gamma > 0$ and $1 + \gamma r > 0$, gives

$$\sqrt{-\sigma''(r)} = \frac{\gamma}{1+\gamma r}. \tag{18}$$

It follows that

$$1 - r\sqrt{-\sigma''(r)} = 1 - \frac{\gamma r}{1+\gamma r} = \frac{1}{1+\gamma r} > 0.$$

Substitution into (16) returns γ. Combining (18) with (4) gives (17).

Q.E.D.

PROPOSITION 4

The same γ is recovered by

$$\int_0^\infty -\sigma''(r)\,dr = \gamma. \tag{19}$$

PROOF.

Using (5),

$$\begin{aligned}\int_0^\infty -\sigma''(r)\,dr &= \int_0^\infty \frac{\gamma^2}{(1+\gamma r)^2}\,dr \\ &= \left[-\frac{\gamma}{1+\gamma r}\right]_0^\infty \\ &= \gamma.\end{aligned}$$

Equivalently,

$$\int_0^\infty -\sigma''(r)\,dr = \sigma'(0) - \lim_{r\to\infty}\sigma'(r) = (\gamma-\mu) - (-\mu).$$

Q.E.D.

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6 Rigidity

THEOREM 3

Let $u \in C^2(I)$. Assume

$$u''(r) < 0$$

and

$$1 - r\sqrt{-u''(r)} \neq 0$$

on I. Suppose there are constants $\mu > 0$ and $\gamma > \mu$ such that

$$\frac{\sqrt{-u''(r)}}{1 - r\sqrt{-u''(r)}} = \gamma \tag{20}$$

and

$$-u'(r) + \sqrt{-u''(r)} = \mu \tag{21}$$

for every $r \in I$, and suppose

$$\int_0^\infty e^{u(r)}\, dr = 1. \tag{22}$$

then

$$u(r) = \ln\left(\frac{\mu^2}{\mu + \gamma}\right) + \ln(1 + \gamma r) - \mu r \tag{23}$$

for every $r \in I$.

PROOF.

Equation (20) gives

$$\sqrt{-u''(r)} = \gamma\left(1 - r\sqrt{-u''(r)}\right).$$

Collecting the terms containing the square root gives

$$\sqrt{-u''(r)} = \frac{\gamma}{1+\gamma r}. \tag{24}$$

The denominator is positive because $r \in I$. Substituting (24) into (21) yields

$$u'(r) = \frac{\gamma}{1+\gamma r} - \mu.$$

Integration gives

$$u(r) = u(0) + \ln(1+\gamma r) - \mu r.$$

Normalization now gives

$$\begin{aligned} 1 &= e^{u(0)} \int_0^\infty (1+\gamma r)e^{-\mu r}\, dr \\ &= e^{u(0)} \frac{\mu+\gamma}{\mu^2}. \end{aligned}$$

Normalization fixes the value

$$u(0) = \ln\left(\frac{\mu^2}{\mu + \gamma}\right),$$

which gives $u = \sigma$ on I.

Q.E.D.

COROLLARY 3

Let u satisfy the differential hypotheses of Theorem 3. The recovery identities (20) and (21), together with normalization (22), are an exact characterization:

$$u = \sigma \text{ on } I \quad \Longleftrightarrow \quad \text{(20), (21), and (22) hold on their stated domains.}$$

PROOF.

If $u = \sigma$ on I, Theorem 2 gives (20) and (21), while Proposition 1 gives (22). Conversely, Theorem 3 gives $u = \sigma$ on I.

Q.E.D.

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7 Parameter uniqueness

THEOREM 4

Let (μ_1, γ_1) and (μ_2, γ_2) satisfy

$$\mu_i > 0, \qquad \gamma_i > \mu_i, \qquad i \in \{1, 2\}.$$

Suppose their canonical exponential representations agree on a nonempty open interval contained in

$$\left(-\frac{1}{\gamma_1}, \infty\right) \cap \left(-\frac{1}{\gamma_2}, \infty\right).$$

Then

$$\mu_1 = \mu_2, \qquad \gamma_1 = \gamma_2.$$

PROOF.

Injectivity of the exponential function gives equality of the logarithmic representations. Equality of their second derivatives gives

$$\frac{\gamma_1^2}{(1+\gamma_1 r)^2} = \frac{\gamma_2^2}{(1+\gamma_2 r)^2}.$$

All quantities in the denominators are positive on the common interval. Taking positive square roots gives

$$\frac{\gamma_1}{1+\gamma_1 r} = \frac{\gamma_2}{1+\gamma_2 r}.$$

Cross-multiplication yields

$$\gamma_1 + \gamma_1\gamma_2 r = \gamma_2 + \gamma_1\gamma_2 r,$$

so

$$\gamma_1 = \gamma_2.$$

Equality of the first derivatives then gives

$$\mu_1 = \mu_2.$$

Q.E.D.

8 Finite differential exhaustion

For a fixed integer $k \geq 0$, let O be a fixed smooth function of the arguments displayed below. A finite-order local differential expression has the form

$$O\Big(r; u(r), u'(r), \ldots, u^{(k)}(r)\Big),$$

THEOREM 5

Let u satisfy the hypotheses of Theorem 3. For every fixed integer $k \geq 0$, every finite-order local differential expression

$$O\Big(r; u(r), u'(r), \ldots, u^{(k)}(r)\Big)$$

is determined entirely by r, μ, and γ. No additional finite local differential data remain.

PROOF.

Rigidity gives $u = \sigma$ on I. Equations (1), (4), and the explicit derivative formula (6) determine every displayed argument from r, μ, and γ. Substitution into O proves the statement.

Q.E.D.

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9 Exact scale invariance

In this section only, display the parameter dependence as

$$\sigma(r; \mu, \gamma).$$

THEOREM 6

For every $\lambda > 0$ and $r > -1/\gamma$,

$$\sigma\left(\frac{r}{\lambda}; \lambda\mu, \lambda\gamma\right) = \sigma(r; \mu, \gamma) + \ln \lambda. \tag{25}$$

For $r > -1/(\lambda\gamma)$, equivalently,

$$\sigma(r; \lambda\mu, \lambda\gamma) = \sigma(\lambda r; \mu, \gamma) + \ln \lambda. \tag{26}$$

PROOF.

Using (1),

$$\begin{aligned}\sigma(r;\lambda\mu,\lambda\gamma) &= \ln\left(\frac{(\lambda\mu)^2}{\lambda\mu+\lambda\gamma}\right)+\ln(1+\lambda\gamma r)-\lambda\mu r\\ &= \ln\lambda+\ln\left(\frac{\mu^2}{\mu+\gamma}\right)+\ln(1+\gamma(\lambda r))-\mu(\lambda r)\\ &= \sigma(\lambda r;\mu,\gamma)+\ln\lambda.\end{aligned}$$

Replacing r by r/λ gives (25).

Q.E.D.

COROLLARY 4

For $r > -1/(\lambda\gamma)$, the exponential representation satisfies

$$e^{\sigma(r;\lambda\mu,\lambda\gamma)} = \lambda e^{\sigma(\lambda r;\mu,\gamma)}. \tag{27}$$

For $r > -1/\gamma$, the normalized differential identity is

$$e^{\sigma(r/\lambda;\lambda\mu,\lambda\gamma)}\, d\left(\frac{r}{\lambda}\right) = e^{\sigma(r;\mu,\gamma)}\, dr. \tag{28}$$

Closure scales according to

$$r_*(\lambda\mu, \lambda\gamma) = \frac{r_*(\mu, \gamma)}{\lambda}. \tag{29}$$

Successive positive scale transformations by λ_1 and λ_2 equal the direct transformation by $\lambda_1\lambda_2$.

PROOF.

Equation (27) is the exponential of (26). Exponentiating (25) and multiplying by $d(r/\lambda) = dr/\lambda$ gives (28). Equation (10) evaluated at $\lambda\mu$ and $\lambda\gamma$ gives (29). For $\lambda_1, \lambda_2 > 0$, two applications of (26) give

$$\begin{aligned}\sigma(r; \lambda_1\lambda_2\mu, \lambda_1\lambda_2\gamma) &= \sigma(\lambda_2 r; \lambda_1\mu, \lambda_1\gamma) + \ln\lambda_2 \\ &= \sigma(\lambda_1\lambda_2 r; \mu, \gamma) + \ln\lambda_1 + \ln\lambda_2 \\ &= \sigma(\lambda_1\lambda_2 r; \mu, \gamma) + \ln(\lambda_1\lambda_2).\end{aligned}$$

This is the direct transformation by $\lambda_1\lambda_2$.

Q.E.D.

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10 Transform recovery

THEOREM 7

For a complex variable s with $\mathrm{Re}(s) > -\mu$,

$$\int_0^\infty e^{-sr} e^{\sigma(r)}\, dr = \frac{\mu^2}{\mu+\gamma} \frac{s+\mu+\gamma}{(s+\mu)^2}. \tag{30}$$

Its meromorphic continuation has a double pole at $s = -\mu$ and a zero at $s = -(\mu + \gamma)$. The pole recovers μ, and the pole-zero separation recovers γ.

PROOF.

Using (2),

$$\begin{aligned}\int_0^\infty e^{-sr} e^{\sigma(r)}\, dr &= \frac{\mu^2}{\mu+\gamma} \int_0^\infty (1+\gamma r) e^{-(s+\mu)r}\, dr \\ &= \frac{\mu^2}{\mu+\gamma}\left(\frac{1}{s+\mu} + \frac{\gamma}{(s+\mu)^2}\right) \\ &= \frac{\mu^2}{\mu+\gamma} \frac{s+\mu+\gamma}{(s+\mu)^2}.\end{aligned}$$

At $s = -\mu$ the numerator equals $\gamma > 0$, so the pole is double. The numerator vanishes at $s = -(\mu + \gamma)$. Their separation is γ.

Q.E.D.

On $[-1/e, 0)$, W_0 is the real Lambert branch with values in $[-1, 0)$, and W_{-1} is the real Lambert branch with values in $(-\infty, -1]$.

THEOREM 8

Let

$$0 < y \le e^{\sigma(r_*)}.$$

The real solutions of

$$e^{\sigma(r)} = y \tag{31}$$

on I are

$$r = -\frac{1}{\mu} W_k\left(-\frac{\mu}{\gamma}\frac{y}{e^{\sigma(0)}}e^{-\mu/\gamma}\right) - \frac{1}{\gamma}, \qquad k \in \{0, -1\}. \tag{32}$$

For $0 < y < e^{\sigma(r_*)}$, W_0 gives the solution below r_*, and W_{-1} gives the solution above r_*. At $y = e^{\sigma(r_*)}$, the branches meet at r_*.

PROOF.

Equation (31) gives

$$\frac{y}{e^{\sigma(0)}} = (1+\gamma r)e^{-\mu r}.$$

Equivalently,

$$-\frac{\mu}{\gamma}\frac{y}{e^{\sigma(0)}}e^{-\mu/\gamma} = -\frac{\mu}{\gamma}(1+\gamma r)\exp\left[-\frac{\mu}{\gamma}(1+\gamma r)\right].$$

Applying W_k and solving for r gives (32). At closure, (15) makes the Lambert argument $-1/e$. For arguments in $(-1/e, 0)$, W_0 lies in $(-1, 0)$ and W_{-1} lies below -1, which gives the stated positions relative to r_*.

Q.E.D.

11 Dimensional identity and self-closure

Let $x > 0$, set

$$r = x^2,$$

and retain

$$f(x) = e^{\sigma(x^2)/2}. \tag{33}$$

THEOREM 9 — DIMENSIONAL SELF-CLOSURE

For every positive integer D,

$$\frac{f''(x)}{f(x)} + \frac{D-1}{x}\frac{f'(x)}{f(x)} = r\big(\sigma'(r)\big)^2 + 2r\sigma''(r) + D\sigma'(r). \tag{34}$$

Also,

$$\frac{(x^{(D-1)/2}f(x))''}{x^{(D-1)/2}f(x)} - \left[\frac{f''(x)}{f(x)} + \frac{D-1}{x}\frac{f'(x)}{f(x)}\right] = \frac{(D-1)(D-3)}{4x^2}. \tag{35}$$

Exact dimensional self-closure is the absence of a residual inverse-square term:

$$\frac{(x^{(D-1)/2}f(x))''}{x^{(D-1)/2}f(x)} = \frac{f''(x)}{f(x)} + \frac{D-1}{x}\frac{f'(x)}{f(x)} \qquad (x > 0).$$

The dimensional identity then gives

$$(D-1)(D-3) = 0.$$

The solutions are

$$D = 1 \qquad \text{or} \qquad D = 3. \tag{36}$$

Among integer spatial dimensions $D \geq 2$, the unique solution is

$$D = 3. \tag{37}$$

PROOF.

Equation (33) gives

$$\frac{f'(x)}{f(x)} = x\sigma'(r).$$

Differentiating and using $r = x^2$ gives

$$\frac{f''(x)}{f(x)} = \sigma'(r) + 2r\sigma''(r) + r\big(\sigma'(r)\big)^2.$$

Adding

$$\frac{D-1}{x}\frac{f'(x)}{f(x)} = (D-1)\sigma'(r)$$

gives (34). Direct differentiation gives

$$\frac{(x^{(D-1)/2}f(x))''}{x^{(D-1)/2}f(x)} = \frac{f''(x)}{f(x)} + \frac{D-1}{x}\frac{f'(x)}{f(x)} + \frac{(D-1)(D-3)}{4x^2},$$

which gives (35). Exact dimensional self-closure holds precisely when $(D - 1)(D - 3) = 0$. Its integer solutions are (36), and restriction to $D \geq 2$ gives (37).

Q.E.D.

12 Three-dimensional spherical closure

For $D = 3$, let

$$r = x_1^2 + x_2^2 + x_3^2. \tag{38}$$

THEOREM 10

The spatial evaluation

$$\sigma(x_1^2 + x_2^2 + x_3^2)$$

has the complete global maximizing set

$$x_1^2 + x_2^2 + x_3^2 = r_* = \frac{1}{\mu} - \frac{1}{\gamma}. \tag{39}$$

The radius is

$$\sqrt{\frac{1}{\mu} - \frac{1}{\gamma}}. \tag{40}$$

Write

$$\mathbf{x} = (x_1, x_2, x_3)^\mathsf{T}, \qquad \|\mathbf{x}\|^2 = r.$$

The spatial gradient is

$$\nabla\sigma(\|\mathbf{x}\|^2) = 2\mathbf{x}\,\sigma'(r). \tag{41}$$

With I_3 the 3×3 identity matrix, the Hessian is

$$\nabla^2\sigma(\|\mathbf{x}\|^2) = 2\sigma'(r)I_3 + 4\sigma''(r)\mathbf{x}\mathbf{x}^\mathsf{T}, \qquad r = \|\mathbf{x}\|^2. \tag{42}$$

At the origin,

$$\nabla^2\sigma(\|\mathbf{x}\|^2)\big|_{\mathbf{x}=0} = 2(\gamma - \mu)I_3 > 0. \tag{43}$$

The origin is a strict local minimum and is not closure. On the closure sphere,

$$\sigma'(r_*) = 0.$$

The Hessian has two zero tangential eigenvalues. Its radial eigenvalue is

$$4r_*\sigma''(r_*) = -4\mu^2 r_* < 0. \tag{44}$$

The sphere is the complete global maximizing set. The spatial evaluation is constant in the tangential directions and strictly maximal in the radial direction.

PROOF.

The map

$$(x_1, x_2, x_3) \longmapsto x_1^2 + x_2^2 + x_3^2$$

has image $[0, \infty)$. Theorem 1 gives the unique maximum at $r = r_*$, so its complete spatial preimage is (39), with radius (40). The chain rule gives (41), and a second differentiation gives (42). At the origin, $\sigma'(0) = \gamma - \mu$, which gives (43). On the closure sphere $\sigma'(r_*) = 0$, so the two tangential eigenvalues vanish. Equation (12) gives the radial eigenvalue (44), which is negative because $r_* > 0$.

Q.E.D.

13 Theorem — Complete closure

THEOREM 11

Let

$$\mu > 0, \qquad \gamma > \mu, \qquad I = \left(-\frac{1}{\gamma}, \infty\right),$$

and let

$$\sigma(r) = \ln\left(\frac{\mu^2}{\mu + \gamma}\right) + \ln(1 + \gamma r) - \mu r.$$

The same uniquely determined σ simultaneously satisfies all eleven conclusions below.

1. Its normalization is unique, and no independent amplitude remains.
2. Its derivative is explicit at every finite order.
3. Its parameters μ and γ are recovered pointwise on I.
4. Recovery and normalization characterize σ exactly by rigidity.
5. Its canonical parameters are unique.
6. Every finite local differential expression is exhausted by r, μ, and γ.

7. It has exact scale invariance, and successive positive scale transformations close under multiplication.

8. Its Laplace transform recovers μ and γ globally.

9. Its exact Lambert inversion has two real branches that meet at closure.

10. Under dimensional self-closure, $D = 3$ is the unique integer spatial dimension $D \geq 2$.

11. In three dimensions, its complete closure set is the sphere (39).

PROOF.

Proposition 1 and equation (3) establish clause 1. Propositions 2 and 3, with equations (4)–(9), establish clause 2. Theorem 2 and Proposition 4, with equations (16)–(19), establish clause 3. Theorem 3 and Corollary 3, with equations (20)–(24), establish clause 4. Theorem 4 establishes clause 5. Theorem 5 establishes clause 6. Theorem 6 and Corollary 4, with equations (25)–(29), establish clause 7. Theorem 7 and equation (30) establish clause 8. Theorem 8 and equations (31)–(32) establish clause 9. Theorem 9 and equations (33)–(37) establish clause 10. Theorem 10 and equations (38)–(44) establish clause 11.

Q.E.D.

14 Exhaustion of independent data

COROLLARY 5

- Normalization fixes the amplitude.
- The local second-order structure recovers μ and γ.
- Rigidity fixes the complete analytic expression.
- The analytic expression fixes every finite derivative.
- The scale law fixes every positive rescaling.
- The transform returns the same parameters globally.
- Dimensional self-closure fixes $D = 3$ among integer spatial dimensions $D \geq 2$.
- Three-dimensional spatial closure fixes the sphere.

No amplitude, parameter information, finite local differential structure, scale prescription, spatial dimension, or closure geometry remains independently specifiable.

> PROOF.
>
> These are clauses 1, 3, 4, 2, 7, 8, 10, and 11 of Theorem 11, respectively.
>
> Q.E.D.